REVIEWS

The Theory of Hydrodynamic Stability, by C. C. LIN. Cambridge University Press, 1955. 155 pp. 22*s*. 6*d*. or $4.25.

Hydrodynamic stability is no newcomer to the literature of fluid mechanics. It was studied by many of the 19th century scientists, who recognized its important part in the neat structure we now call classical hydrodynamics. The question of instability in vortex sheets was considered by Stokes and Helmholtz, and a number of other special problems were dealt with successfully. It is instructive to recall also the care and skill with which scientists of the last century examined the general question of the modes of disturbance possible in a continuous dynamical system. Intuitive concepts of stability are not easily extended to a continuous medium in motion, and a contemporary physicist regarding the present subject from a non-expert's viewpoint might still attach foremost importance to this basic issue. Thus there may be some who would regret that the book under review says very little about it. This is scarcely cause for complaint, however, for the book is admittedly preoccupied with the intricate mathematics of modern theories.

In this as in so many other subjects, the first definite steps towards the modern science were made by Lord Rayleigh, who attacked the general problem of the stability of plane flows in which the velocity and vorticity have a continuous distribution. His work culminated in the well-known theorem which states that a flow is stable when its velocity profile possesses no point of inflexion. It is interesting that he commenced this work a few years before Reynolds made his important experiments first revealing transition to turbulence in a pipe, although this discovery naturally encouraged him in his later researches. (Strangely enough, a critical Reynolds number for Poiseuille flow has not been established theoretically even to this day.) The problem of explaining the mechanism of transition is still the chief stimulus to research in hydrodynamic stability, and our answers to the crucial question how random turbulent fluctuations develop from oscillatory laminar flows remain no more than tentative. Nevertheless, in those cases where the initial laminar flow is only slightly disturbed and the solid boundaries are smooth, an essential first stage in the process is without doubt an instability, which results in the amplification of oscillations whose wave numbers are contained within certain bands.

The confliction of the inferences from Rayleigh's theorem and from Reynold's experiments led the earlier workers to the conclusion that viscosity plays an essential part in the development of instability; and, after several unsuccessful attempts to explain transition by reference to the energy of the disturbances, the basis of the modern theory was laid by Orr and Sommerfeld, who derived the equation governing the propagation of small wavy disturbances in otherwise steady laminar flow. Subsequent work on the solution of this equation was principally directed, on the one hand, by G. I. Taylor to the flow between concentric rotating cylinders and, on the other, by Heisenberg to the much more difficult case of two-dimensional Poiseuille flow. Taylor performed experiments which substantiated his theoretical results with extraordinary completeness; but Heisenberg's analysis contained certain mathematical obscurities, and, besides the lack of any experimental support, his conclusions were contradicted by other investigators. The extension, by Tollmien and Schlichting, of Heisenberg's analytical methods to the case of boundary-layer stability carried with it the doubts attached to his earlier work, and was also subject to the criticism that the growth of the boundary layer had been neglected without wholly satisfactory justification. The controversy led to a decided schism between workers in Germany, where development of stability theory continued, and those in other countries who preferred an alternative theory proposed by Taylor. The conflict was finally resolved in favour of stability theory—at least, in circumstances where the disturbance of the laminar regime is small—by the experiments of Schubauer and Skramstad. Their investigation revealed boundary-layer oscillations whose structure was in remarkably good agreement with the predictions of Schlichting, for which further confirmatory evidence was subsequently found by Liepmann. The experimental vindication of stability theory eventually led Lin to reconsider the mathematical foundation of the subject, which he was able to clarify in large measure. Indeed, the present status of the theory owes a great deal to Lin's efforts.

There can be no reasonable doubt about the usefulness of Professor Lin's monograph, for it is the first major survey of the theory of hydrodynamic stability to appear in book form. For a complete account of the physical aspects of the subject one must look elsewhere; yet within its clearly defined scope the book enables the reader to trace all the main achievement of the mathematical theory without becoming unduly immersed in the tortuous arguments that accompanied its development. Its success as an exposition can perhaps best be judged if we refer to certain of the more subtle aspects of the theory.

Regarding the ticklish problem of boundary-layer stability, it is apparent that viscosity must play an unusual role. Whereas Rayleigh's theorem states that the absence of points of inflexion from the velocity profile guarantees the stability of an inviscid flow, experiment has established that real flows become unstable at sufficiently high Reynolds numbers whatever their velocity profiles. Intuition would suggest that viscosity is essentially dissipative, and therefore provides a mechanism for damping out any small oscillation that might be present. Such an action is indeed effective in some instances, as in Taylor's problem of flow between rotating cylinders. However, in boundary-layer flow, which for our purpose may be considered as any parallel or nearly-parallel flow, the effect of viscosity is not so simple; for by some means it promotes the transfer of energy from the basic laminar flow to the disturbance, which is thereby amplified.

Speaking in physical terms, the dual action of viscosity is connected with the fact that its effects are diffusive as well as dissipative in character, and consequently can assist in spreading any local concentration of vorticity to neighbouring parts of the fluid. This diffusive action is certainly relevant, since the theory shows that the supposed regions of concentrated vorticity do exist in the disturbed flow. The mathematical description of these regions presents severe difficulties, although the manner in which they emerge from the solution of the equation of disturbed motion can be appreciated without the need of full attention to the intricate mathematics involved. Professor Lin appears to have planned his monograph with didactic aims such as this in view; the major part of the text is devoted to general principles illustrated by specific examples, and a detailed discussion of the finer mathematical points arising from the boundary-layer solutions is deferred to the final chapter. This is a sensible arrangement; in addition to possessing a logical appeal, it surely widens the class of readers to which the book may be of interest or use.

As an introduction to the technique for solving problems of hydrodynamic stability, the author chooses to concentrate on two examples in which definite conclusions about the critical Reynolds number have been reached; these are Couette flow between rotating cylinders, and plane Poiseuille flow, the archetype for problems of boundary-layer stability. In the former example, centrifugal forces in the fluid are prodominant in determining the limits of stability; in the latter, the essential effect is a balance between the diffusive and dissipative actions of viscosity. In his introduction (p. 11) the author remarks, " . . . how easily problems of hydrodynamic stability can be formulated as definite characteristic-value problems ". The reader soon afterwards discovers that in general detailed solutions cannot be obtained with comparable ease. A startling illustration of the magnitude of the difficulty is L. H. Thomas's estimate, quoted in the monograph, of the working time required for the solution of the plane Poiseuille problem; for a strictly limited set of numerical results, a high-speed numerical calculator was occupied for the equivalent of two weeks continuous running, the estimated equivalent of " 100 years work by hand computation ".

Having explained clearly the principal methods of analysis, the author discusses certain general physical aspects of the theory, in particular, the conclusions reached by Rayleigh concerning the stability of an inviscid fluid. The practical implications of the instability associated with inflexional velocity profiles are important and numerous. The characteristic dynamical behaviour of fluid in the neighbourhood of a region of maximum vorticity persists throughout the range of Reynolds number for which instability can occur, with the result that the critical Reynolds number is lower and the range of wave-numbers containing oscillations which suffer amplification is wider than those for non-inflexional profiles. Accordingly, the existence of an inflexional velocity profile can often be identified with a more powerful tendency to instability—a straightforward observation, but one which enables qualitative explanations to be given for such diverse phenomena as the instability that originates in the flow near the walls of a wind tunnel in which turbulence-damping screens of high resistance are placed at the entry to the contraction, the low value of the Reynolds number at which transition occurs in the free-convective flow near a heated vertical plate, and the premature instability of the laminar boundary layer on a sweptback wing.

In the remaining chapters, excellent accounts are given of the known results of stability calculations for the laminar boundary layer on a flat plate, including the work of Lees and Lin on the compressible boundary layer, and for the boundary layer on a porous surface. In this latter case, which is of considerable aeronautical interest, experimental work has provided remarkable confirmation of some of the deductions from the theory, especially the magnitude of the rate of flow into the surface required to ensure stability of the boundary layer at a given Reynolds number. A separate chapter is devoted to a concise description of the application of stability theory to some meteorological and astrophysical problems. The final chapter deals with the mathematical difficulties inherent in the theory of boundary layer stability, and is based very broadly on Professor Lin's own contribution to the subject. It is a model of clarity in view of the delicate arguments involved, and concentrates on essential matters of principle. This chapter alone could ensure the permanence of the monograph as a guide to the theory.

The work as a whole bears the stamp of authority. Its style is characterized by an economy which will commend it to many, but offers little to encourage others who may find the going hard. The reading of it to full advantage demands considerable effort, but any blame for this lies with the difficulties of the subject, not with the author. Workers in fluid mechanics will be indebted to Professor Lin for presenting the theory and many of its applications in so compact a form.

P. R. Owen